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A note on the interpretation of seismic surface waves over 3-d structures

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SUMMARY

In the two-dimensional acoustic case, the sound velocity of a heterogeneous medium can be determined from the amplitude and phase of almost any wavefield (but not from its phase alone) by inserting the wavefield into the Helmholtz equation and resolving for the wavenumber. Seismic surface waves in a laterally heterogeneous halfspace do not obey the Helmholtz equation; however when the heterogeneity is smooth, the Helmholtz equation is approximately valid for each mode separately. Using synthetic multi-mode wavefields, we investigate whether the same equation can be used to improve the structural interpretation of seismic surface waves. We find that the Helmholtz equation images lateral heterogeneity much better than the eikonal equation on which the conventional interpretation of surface-wave dispersion is based.

Key words: phase velocity, structural interpretation of phase velocities, surface waves

1 THE PROBLEM

"Interferences" between multipathing seismic surface waves have for long annoyed seismologists. They cause the observed phase velocities to scatter around those expected from the properties of the medium, and although the scatter may be reduced by averaging, there is no guarantee that the average is unbiased.

Behind the practical problem of interfering waves is a simple mathematical fact: a single plane wave in a homogeneous and isotropic medium propagates with a phase velocity characteristic for the medium, but a superposition of such waves (a nonplane wave) propagates with a velocity that also depends on the distribution of amplitudes. The phase velocity of a nonplane wave does therefore not map the properties of the underlying medium correctly. In inhomogeneous media, the wavefield is only accidentally plane in certain isolated points (normally the saddle-points of the amplitude). How, then, can the phase velocity of surface waves be interpreted in terms of the structure of the medium?

2 SURFACE WAVES IN A LATERALLY HOMOGENEOUS HALFSPACE

When we restrict ourselves to isolated modes in a horizontally stratified medium, a clear answer is at hand. As a matter of convenience, we will consider wavenumbers in place of phase velocities. The wavenumber of an arbitrary wave will be called *dynamic* and that of a locally plane wave *structural* (because it is determined by the velocity-depth structure of the medium alone). The dynamic wavenumber vector $\vec{w}$ of a wavefield $F(x, y, \omega)$ is the negative gradient of its phase, thus

$$\vec{w} = -\nabla \Im \ln F \tag{1}$$

where $\Im$ denotes the imaginary part. The structural wavenumber k is the one that appears in the Helmholtz equation $(\nabla^2 + k^2)F = 0$. Its square can therefore be written as

$$k^2 = -F^{-1}\nabla^2 F \tag{2}$$

The two wavenumbers $w = |\vec{w}|$ and k are in general different but can be converted into each other when the distribution of amplitudes around the point of observation is known. Let $a = \Re \ln F$ be the logarithmic amplitude of the wavefield, then

$$k^2 = w^2 - (\nabla a)^2 - \nabla^2 a \tag{3}$$

A derivation and a detailed discussion are found in Wielandt(1993). It is also shown there that using dynamic phase velocities for a structural interpretation means approximating the wave equation by the eikonal equation.

The correction terms in eq. (3) can be substantial even under optimistic assumptions on the wavefield. Figure 1 shows a synthetic wavefield in a medium that is homogeneous in the area of the figure but supposed to contain slight heterogeneities outside this area. We model this situation with a "direct" plane wave of unit amplitude and 48 weak "scattered" plane waves incident from all directions with random amplitudes and phases. The total energy of the scattered waves is only 0.11% of the direct wave. Certainly most seismologists would consider this as negligible. However, the dynamic phase velocity (lower left panel) varies between 3.72 and 4.24 km/s in the area of the figure. Thus, 0.11% of scattered energy can produce variations of the phase velocity as large as those caused by different tectonic structures (Knopoff, 1972). The lower right panel shows that the application of eq. (3) completely removes the spurious "anomalies" of the phase velocity.

The relative scatter of the amplitude, $\delta a/a$, and that of the dynamic phase velocity, $\delta c/c$, are roughly the same in fig. 1. This is not coincidental. It can be shown (in a linear approximation) that plane waves incident under the angle φ against the direct wave cause the dynamic phase velocity to

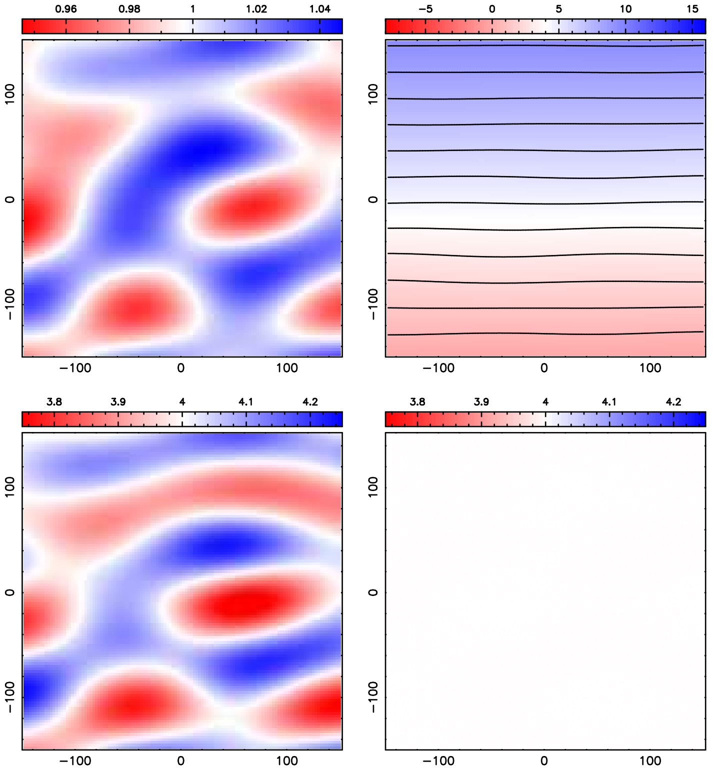

Figure 1. A numerical experiment with a nonplane wave in a two-dimensional homogeneous medium. The wavefield consists of a "direct" plane wave incident from the top and 48 "scattered" plane waves incident from all directions with small, randomly chosen amplitudes. The structural phase velocity (the phase velocity of each plane wave) is 4 km/s and the wavelength 200 km. Axes are labelled in km; the area is 300 x 300 km. Top left: the total amplitude of the wavefield. Top right: the phase in radians, with 8 isolines per wavelength. Bottom left: the dynamic phase velocity (the local phase velocity of the total field) in km/s. Bottom right: the structural phase velocity as calculated from eq. (3), in the same scale as the dynamic velocity.

scatter around the structural velocity by $\delta c/c = 2(\sin\frac{\varphi}{2})^2\,\delta a/a$. When φ is a right angle, c and a scatter by the same percentage.

The figure represents a situation where a substantial portion of the scattered waves propagate backward through the area of investigation, creating an interference pattern with nearly half of the original wavelength. Such a wavefield would have to be sampled at intervals of less than a quarter wavelength in order to calculate the correction terms in eq. (3). This may however not be typical for seismic surface waves. Normally, the percentage of scattered energy will be much higher but most of the scattered waves will travel in the forward direction. The interference pattern will then be stronger but smoother while its effect on the phase velocity may be of the same order. We conclude this from theoretical considerations and from amplitude and phase patterns observed in the German Regional Seismic Network (Friederich et al., 1994).

The difference between the structural and dynamic wavenumbers as well as the corresponding residuals of the phase travel-time oscillate with the interference pattern. Travel-time residuals caused by interference do not accumulate over large propagation paths, in contrast to the polar phase shift which is implicit in eq. (3) and must separately be taken into account when eq. (3) is not used. The effect of correcting globally measured phase velocities with eq. (3) is therefore probably small. However, the travel-time residuals do not average out over large paths, and therefore cannot be ignored in a tomographic inversion.

The interfering waves in fig. 1 can be thought of as being caused by heterogeneous structure outside the area. The figure thus demonstrates that with eq. (3) we can eliminate the effect of such a structure on phase-velocity measurements. We now investigate how well a structural heterogeneity within the area can be mapped with seismic surface waves.

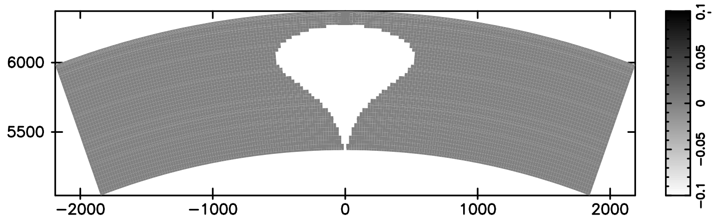

Figure 2. A vertical cross-section of the test structure in the top 1000 km of the mantle. The maximum diameter is 667 km, the depth of burial 50 km, and the anomaly of the shear-wave velocity is -10%. The scales show rectangular coordinates in km.

3 GENERALIZATION TO THE LATERALLY HETEROGENEOUS HALFSPACE

Surface waves in a laterally heterogeneous halfspace do not obey the Helmholtz equation, even when only one mode at a time is considered. In fact the concept of modes, and of surface waves as a distinct solution of the elastic wave equation, breaks down in such a medium. Phase velocities can of course still be measured along the surface, but in contrast to the 2-d acoustic or 3-d laterally homogeneous case, the *quasi-structural* velocities obtained from eq. (3) will differ from one event to another (although much less than the dynamic velocities). The interpretation of phase-velocity maps, even of the quasi-structural type, is not a well-posed problem. Another more practical difficulty is that seismic data from which the second spatial derivatives of the amplitude can be estimated with some confidence are normally unavailable.

Nevertheless the approximation of seismic surface waves by acoustic waves is a useful concept. The approximation consists in neglecting mode conversion and the directivity of scattering from a point heterogeneity; these two simplifications can be shown to restore the validity of the Helmholtz equation. The quasi-structural phase velocity is in practice not obtained from eq. (3) but from a multi-event inversion procedure (Friederich, 1998) in which the observed wavefields in a finite number of observation points are modelled as 2-d acoustic waves. The inversion adjusts both the quasi-structural velocity and the shape of each incident wavefield so that the synthetic wavefield fits the vertical component of the seismic wavefield in amplitude and phase at all stations. In this way we can obtain from the seismic data a set of maps of the quasi-structural phase velocity without explicitly differentiating observed wavefields.

We now show that the structure at depth can be reconstructed reasonably well from maps of the quasi-structural phase velocity. The procedure is as follows: we compute wavefields for regional 3-d perturbations of a spherically symmetric earth model. We use a multi-mode, multiple-scattering algorithm (Friederich, 1999); its accuracy has been checked for spherically symmetric perturbations, which are most difficult to handle by scattering. The background model in which the incident (reference) field propagates is the Preliminary Reference Earth Model (PREM) by Dziewonski and Anderson (1981). The incident wave is a fundamental-mode, circular surface wave from an explosive point source at 19 degrees epicentral distance. The perturbation that generates the scattered wave is a mushroom-shaped "plume" between 50 and 1000 km depth (figure 2). This structure causes mode conversion; we compute the total wavefield as a superposition of all Rayleigh modes with phase velocities lower than 10 km/s. Love modes are not included because they do not contribute to the vertical component which we evaluate; we neglect however those components of the wavefield that have been doubly converted from P-SV to SH polarization and back. We then use equations (1) and (3) to generate maps of the lo-

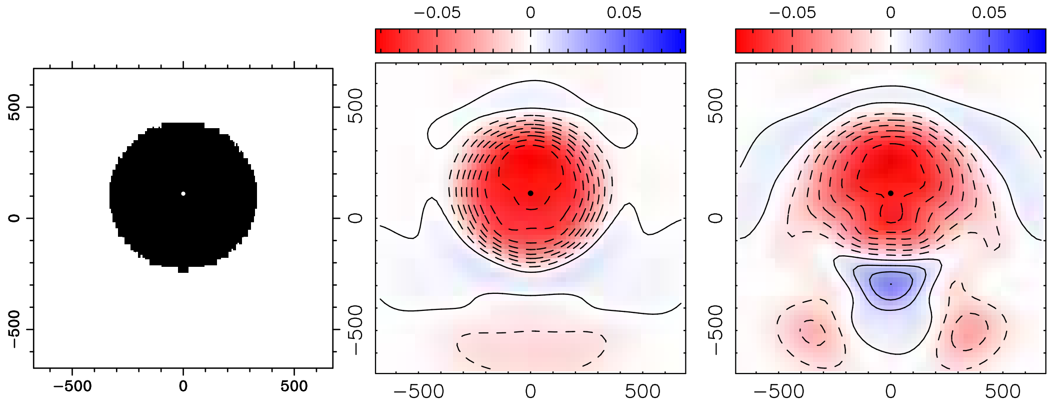

Figure 3. Left: the largest horizontal cross-section of the plume at a depth of about 200 km. Center: the quasi-structural phase velocity (as defined by eq. 3) at 10 mHz; the wavelength is approximately 400 km. Right: the dynamic phase velocity (eq. 1) at 10 mHz. The wavefield propagates downward. Coordinate axes are labelled in km; the greyscale indicates the relative anomaly.

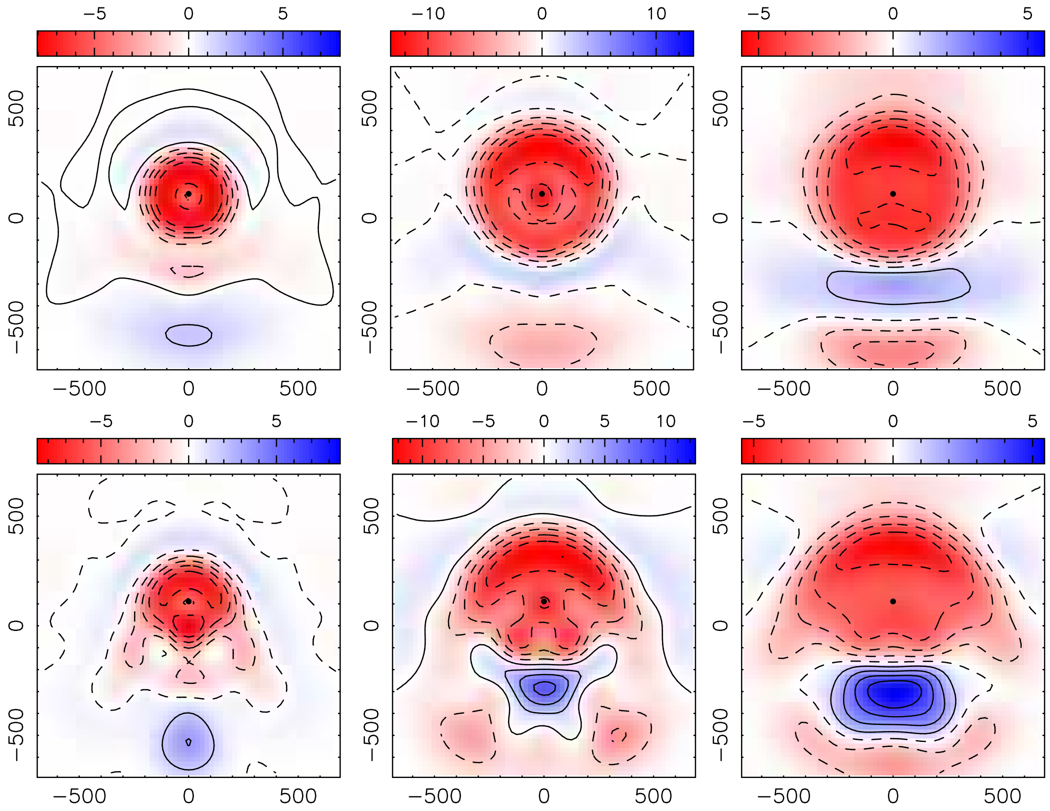

Figure 4. Horizontal sections of the reconstructed plume at depths of 100 km (left), 200 km (center), and 400 km (right). The wavefield propagates from top to bottom. The center of the plume is marked by a dot. The upper panels are based on the quasi-structural velocity and the lower ones on the dynamic velocity. Coordinate axes are labelled in km; the greyscale is in percent.

cal dynamic and quasi-structural phase velocities at different frequencies (Hunzinger, 1999). In a second step, we invert each set of maps into a three-dimensional image of the anomaly in the mantle. The inversion is in principle done locally but in order to impose smoothness conditions, the phase-velocity maps are expanded into two-dimensional Hermite-Gauss polynomials, and the frequency-dependent coefficients of the expansion are inverted into depth-dependent ones (Friederich et al., 1994; Friederich & Wielandt, 1995; Friederich, 1998). Horizontal sections of the anomaly are then reconstructed from the inverted coefficients.

4 RESULTS

Figure 3 compares maps of the quasi-structural and dynamic phase velocities at 10 mHz (center and right) with the size and position of the plume (left). The wavefield propagates from top to bottom. The structural velocity images the plume (whose maximum diameter is about 1.5 wavelengths) very well. The dynamic velocity gives a more diffuse image and exhibits spurious anomalies resulting from diffraction in the wake of the structure. It is remarkable that the image of the anomaly in the structural map appears exactly in the correct place. Since the scattered wavefield is dominated by forward scattering, we had originally suspected that the image might be displaced downstream.

Figure 4 shows horizontal sections of the reconstructed plume. The depth is 100 km (left), 200 km (center) and 400 km (right). The upper three panels are based on the quasi-structural phase velocity and the lower ones on the dynamic phase velocity. Frequencies from 6 to 50 mHz were used in the inversion. The initial increase of the plume diameter with depth is reflected in the 100 and 200 km panels. The resolution at 400 km depth is however insufficient to show the tapering out of the diameter at depth; instead, the amplitude of the reconstruction is reduced.

5 CONCLUSION

Structural phase velocities in a strict sense – which might be defined as the phase velocities of local modes (Maupin, 1988) – cannot exactly be determined from locally measured phase velocities of seismic surface waves. Nevertheless the relationship (3) which solves the problem for acoustic waves and for surface waves in laterally homogeneous media is effective both in removing the effect of interfering wavetrains and in significantly reducing diffraction patterns in the reconstructed heterogeneity. The price that must be paid is that the network of stations has to be so dense that the second spatial derivatives of the amplitude of the wavefield can be estimated (although they need not be explicitly determined). The spatial resolution is essentially limited by the station density, not by diffraction (Friederich & Wielandt, 1995). The inversion of raw (dynamic) phase velocities with the same density of station does not result in a correct tomographic image.

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